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Resonant and non-resonant amplitude modulation of Mössbauer gamma-quanta

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Abstract. Resonant/non-resonant amplitude modulation of gamma radiation occurs when it passes through a medium whose resonant/non-resonant dispersive properties are varied periodically with time. These phenomena are described from a unified point of view. Results are understood in the framework of the classical electromagnetic theory.

1. Introduction

Each perturbation affecting the time behaviour of a propagating wave modifies also its spectral distribution. A variety of such phenomena, commonly referred to as wave modulation, has been observed for all wave processes in nature. The Mössbauer effect made possible the investigation of the modulation phenomena occurring with gamma-ray wavepackets. Most of the studies so far have been concerned with the frequency modulation, which appears always when the phase of the wave is changed periodically with the time, in the case of a mutual vibration between the source and the absorber, for instance.

Amplitude modulation (AM) would occur when the wave amplitude is periodically changed for a time comparable with the inverse of the damping constant Γ. The experiments so far are based on two different ways of altering the amplitude. In the first method, suggested by Kamenov (1970) and used by Isaak and Preikschat (1972), Ruby *et al* (1973) and Hauser *et al* (1974), AM is realised by a periodic change in the non-resonant absorption of the medium between the emitting and the absorbing nuclei, chopping the radiation by a fast rotating wheel with a periodic structure, while in the second method, used by Asher *et al* (1974) and particularly by Cashion and Clark (1979), modulation is achieved by a periodic change in the resonant admissability between the source and the absorber, vibrating an intermediate absorber at megahertz frequencies. These phenomena are referred to below as 'non-resonant' and 'resonant' AM of gamma rays, respectively.

A detailed theory of the non-resonant AM is proposed by Wojtowiecki and Sazonov (1975), and a theory of the resonant AM has recently been developed by Tsankov (1980) (this paper will be cited below as I).

The purpose of the present communication is to show that both types of AM are, in fact, two different manifestations of the same phenomenon, which may be adequately described from a unified point of view. An additional advantage of such an approach is the possibility of making a more precise analysis of this phenomenon.

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2. The non-resonant amplitude modulation

For the purposes of the present investigation we shall use an approach, slightly different from that of Wojtowiecki and Sazonov (1975) and rather similar to that of Ruby *et al* (1973).

We assume that the gamma radiation passes through a medium characterised by a complex index of refraction which is periodically changed with a period $T=2\pi/\Omega$. Then the modulated probability amplitude for a single photon is

$$a(t)=\mathrm{e}^{\mathrm{i}\omega_0 t-\Gamma t/2}B(t+\eta), \tag{1}$$

where $B(t)$ is the modulation function. Its pattern depends on the conditions of the concrete experiment. The symbol η denotes the phase of the modulation function at moment $t=0$.

Function $B(t)$ is periodic. Substituting its Fourier series expansion

$$B(t)=\sum_{k=-\infty}^{\infty} c_k\,\mathrm{e}^{\mathrm{i}k\Omega t} \tag{2}$$

in (1), the frequency component corresponding to (1) may be written in the form

$$a(\omega)=\int_0^{\infty}\mathrm{d}t\,\mathrm{e}^{-\mathrm{i}\omega t}a(t)=\sum_{k=-\infty}^{\infty} c_k\,\mathrm{e}^{\mathrm{i}k\Omega\eta}\frac{1}{\Gamma/2+\mathrm{i}(\omega-\omega_0-k\Omega)}. \tag{3}$$

The spectral distribution of the modulated radiation may be obtained by squaring (3) and averaging the result over η:

$$W(\omega)=\sum_{k=-\infty}^{\infty}|c_k|^2\frac{\Gamma^2/4}{(\omega-\omega_0-k\Omega)^2+\Gamma^2/4}. \tag{4}$$

It remains to estimate the coefficients c_k:

$$c_k=\frac{1}{T}\int_0^T \mathrm{d}t\,B(t)\,\mathrm{e}^{-\mathrm{i}k\Omega t}. \tag{5}$$

For the simplest case of rectangle modulation,

$$B(t)=\begin{cases}F_1\,\mathrm{e}^{\mathrm{i}\phi_1} & 0\leqslant t\leqslant\tau\\ F_2\,\mathrm{e}^{\mathrm{i}\phi_2} & \tau<t<T.\end{cases} \tag{6}$$

This form of $B(t)$ corresponds to the investigations of the previous authors. Substitution of (6) into (5) yields

$$\begin{aligned}|c_k|^2&=\frac{\sin^2(k\pi\tau/T)}{k^2\pi^2}[F_1^2+F_2^2-2F_1F_2\cos(\phi_2-\phi_1)], \qquad k\neq 0,\\ |c_0|^2&=F_1^2\left(\frac{\tau}{T}\right)^2+F_2^2\left(1-\frac{\tau}{T}\right)^2+2F_1F_2\frac{\tau}{T}\left(1-\frac{\tau}{T}\right)\cos(\phi_2-\phi_1).\end{aligned} \tag{7}$$

The identity between equations (4) and (7) here and the results of Wojtowiecki and Sazonov may be exactly proved. However, the form of the results derived above has the advantage that each term in the sum (4) corresponds to a separate line. Thus, the spectrum of the AM radiation consists of an infinite set of satellites, equidistantly spaced at $\omega=\omega_0+k\Omega$, with Lorentzian shape and natural width, and intensities $|c_k|^2$ defined by

(7). The area under the spectrum is

$$S=\int_0^{\infty}\mathrm{d}\omega\ W(\omega)=\pi\frac{\Gamma}{2}\sum_{k=-\infty}^{\infty}|c_k|^2=\pi\frac{\Gamma}{2}\left[F_1^2\left(\frac{\tau}{T}\right)+F_2^2\left(1-\frac{\tau}{T}\right)\right],\tag{8}$$

which is expected, if one takes into account that $F_i^2=\mathrm{e}^{-\mu_i d_i}$, $i=1,2$, is the attenuation of gamma radiation behind the modulator. The following identity was used to obtain (8):

$$\sum_{k=1}^{\infty}\frac{\cos kx}{k^2}=\frac{\pi^2}{6}-\frac{\pi x}{2}+\frac{x^2}{4}.\tag{9}$$

3. The resonant amplitude modulation

This phenomenon is observed when the gamma radiation from a source at rest is passed through an absorber (modulator) containing Mössbauer nuclei, which is vibrated at megahertz frequencies, and the spectrum of the transmitted radiation is scanned by a secondary absorber driven by a conventional Mössbauer vibrator. In this case the amplitude modulation is due to the periodic change in the resonant absorption of the modulator.

This phenomenon is theoretically investigated in I. Restricting the following considerations in the most fundamental case, when the resonant frequency of the modulator is not shifted with respect to that of the source, the spectrum of the radiation behind the modulator has the form

$$W(\omega)=\frac{1}{T}\int_0^T\mathrm{d}\eta\ |a(\omega)_\eta|^2,\tag{10}$$

where

$$\begin{aligned}a(\omega)_\eta={}&\mathrm{J}_0(\varkappa A)\sum_{l=-\infty}^{\infty}\mathrm{J}_l(\varkappa A)\,\mathrm{e}^{-\mathrm{i}l\Omega\eta}[\Gamma/2+\mathrm{i}(\omega-\omega_0+l\Omega)]^{-1}\exp\left(-\frac{D\Gamma/4}{\Gamma/2+\mathrm{i}(\omega-\omega_0+l\Omega)}\right)\\&+\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\mathrm{J}_k(\varkappa A)\mathrm{J}_l(\varkappa A)\,\mathrm{e}^{\mathrm{i}(k-l)\Omega\eta}\left[\frac{\exp((\mathrm{i}D\Gamma/4)/k\Omega)}{\Gamma/2+\mathrm{i}[\omega-\omega_0-(k-l)\Omega]}\right.\\&\left.-\sum_{n=1}^{\infty}\left(\frac{\mathrm{i}D\Gamma/4}{k\Omega}\right)^n\sum_{p=0}^{\infty}\frac{(-D\Gamma/4)^p}{(n+p)![\Gamma/2+\mathrm{i}(\omega-\omega_0+l\Omega)]^{p+1}}\right];\end{aligned}\tag{11}$$

here D denotes the effective modulator thickness, A stands for its vibration amplitude, and $\varkappa=2\pi/\lambda$ is the wavenumber of the gamma quantum.

This presentation may be considerably simplified under the assumption that the modulation frequency is much larger than the linewidth Γ: $\Gamma/\Omega\to 0$. Then

$$\begin{aligned}a(\omega)_\eta={}&\sum_{l=-\infty}^{\infty}\mathrm{J}_0(\varkappa A)\mathrm{J}_l(\varkappa A)\,\mathrm{e}^{-\mathrm{i}l\Omega\eta}[\Gamma/2+\mathrm{i}(\omega-\omega_0+l\Omega)]^{-1}\exp\left(-\frac{D\Gamma/4}{\Gamma/2+\mathrm{i}(\omega-\omega_0+l\Omega)}\right)\\&+\sum_{k=-\infty}^{\infty}{}'\sum_{l=-\infty}^{\infty}\mathrm{J}_k(\varkappa A)\mathrm{J}_l(\varkappa A)\,\mathrm{e}^{\mathrm{i}(k-l)\Omega\eta}\frac{1}{\Gamma/2+\mathrm{i}[\omega-\omega_0-(k-l)\Omega]},\end{aligned}\tag{12}$$

or, using the relation

$$\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\mathrm{J}_k(\varkappa A)\mathrm{J}_l(\varkappa A)\,\mathrm{e}^{\mathrm{i}(k-l)\Omega\eta}\frac{1}{\Gamma/2+\mathrm{i}[\omega-\omega_0-(k-l)\Omega]}=\frac{1}{\Gamma/2+\mathrm{i}(\omega-\omega_0)},$$

one obtains

$$a(\omega)_\eta = \frac{1}{\Gamma/2 + \mathrm{i}(\omega - \omega_0)} + \mathrm{J}_0(\varkappa A) \sum_{l=-\infty}^{\infty} \mathrm{J}_l(\varkappa A)\, \mathrm{e}^{-\mathrm{i}l\Omega\eta}$$

$$\times [\Gamma/2 + \mathrm{i}(\omega - \omega_0 + l\Omega)]^{-1} \left[\exp\left(-\frac{D\Gamma/4}{\Gamma/2 + \mathrm{i}(\omega - \omega_0 + l\Omega)}\right) - 1\right]. \qquad (13)$$

The average over η in (10) is now easily carried out, and the result is proportional to

$$W(\omega) = \sum_{k=-\infty}^{\infty} W_k(\omega), \qquad (14)$$

where $W_k(\omega)$ is the profile of the kth line participating in the spectrum (14):

$$W_k(\omega) = \frac{\Gamma^2/4}{(\omega - \omega_0 - k\Omega)^2 + \Gamma^2/4} \mathrm{J}_0^2(\varkappa A) \mathrm{J}_k^2(\varkappa A) \left[1 + \exp\left(-\frac{D\Gamma^2/4}{(\omega - \omega_0 - k\Omega)^2 + \Gamma^2/4}\right)\right.$$

$$\left. -2 \exp\left(-\frac{D\Gamma^2/8}{(\omega - \omega_0 - k\Omega)^2 + \Gamma^2/4}\right) \cos\left(\frac{(\omega - \omega_0 - k\Omega) D\Gamma/4}{(\omega - \omega_0 - k\Omega)^2 + \Gamma^2/4}\right)\right],$$

$$k \neq 0 \qquad (15)$$

$$W_0(\omega) = \frac{\Gamma^2/4}{(\omega - \omega_0) + \Gamma^2/4} \left[[1 - \mathrm{J}_0^2(\varkappa A)]^2 + \mathrm{J}_0^4(\varkappa A) \exp\left(-\frac{D\Gamma^2/4}{(\omega - \omega_0)^2 + \Gamma^2/4}\right)\right.$$

$$\left. + 2[1 - \mathrm{J}_0^2(\varkappa A)] \mathrm{J}_0^2(\varkappa A) \exp\left(-\frac{D\Gamma^2/8}{(\omega - \omega_0)^2 + \Gamma^2/4}\right) \cos\left(\frac{(\omega - \omega_0) D\Gamma/4}{(\omega - \omega_0)^2 + \Gamma^2/4}\right)\right].$$

Expressions (15) make possible a more powerful analysis of the resonant AM of the gamma radiation. Certain properties of the spectrum have already been pointed out in I. An additional feature is that the profile of the sidebands $W_k(\omega)$, $k \neq 0$, is independent of the vibration amplitude $\varkappa A$, which participates in their intensities only, while the profile of the unshifted line $W_0(\omega)$ depends on both the modulator thickness D and $\varkappa A$.

Intensities at maxima in the spectrum may be explicitly written

$$W_{00} = W_0(\omega)|_{\omega=\omega_0} = [1 - \mathrm{J}_0^2(\varkappa A)\,(1 - \mathrm{e}^{-D/2})]^2,$$
$$W_{k0} = W_k(\omega)|_{\omega=\omega_0 + k\Omega} = \mathrm{J}_0^2(\varkappa A) \mathrm{J}_k^2(\varkappa A)(1 - \mathrm{e}^{-D/2})^2, \qquad k \neq 0, \qquad (16)$$

as well as the area under the spectrum after the modulator:

$$S = \int_0^\infty \mathrm{d}\omega\, W(\omega) = \tfrac{1}{2}\pi\Gamma[1 - \mathrm{J}_0^2(\varkappa A)] + \tfrac{1}{2}\pi\Gamma\, \mathrm{e}^{-D/2} \mathrm{I}_0\left(\frac{D}{2}\right) \mathrm{J}_0^2(\varkappa A) \qquad (17)$$

(the integrals are the same as in the paper of Mössbauer and Wiedeman (1960)).

These results may be generalised for the case of incoherent ultrasonic vibrations of the modulator. Then it is necessary to average the vibration amplitude over the Rayleigh distribution (cf Abragam 1964):

$$P(a) = (a/A^2) \exp(-a^2/A^2).$$

So, the spectrum area (17) becomes

$$S_{\mathrm{incoh}} = \tfrac{1}{2}\pi\Gamma[1 - \mathrm{e}^{-\varkappa^2 A^2} \mathrm{I}_0(\varkappa^2 A^2)] + \tfrac{1}{2}\pi\Gamma\, \mathrm{e}^{-D/2} \mathrm{I}_0(D/2)\, \mathrm{e}^{-\varkappa^2 A^2} \mathrm{I}_0(\varkappa^2 A^2). \qquad (18)$$

Formulae (17) and (18) might be used for an experimental evaluation of the coherency degree of the ultrasonic phonons generated in a solid.

4. Connection between the two formalisms

Formulae (4), (7) and (14), (15), related to the spectrum of non-resonant and resonant AM of gamma radiation respectively, are rather close to its structure. This similarity suggests the following correspondence:

$$\tau/T \leftrightarrow 1-\mathrm{J}_0^2(\varkappa A), \qquad F_1 \leftrightarrow 1,$$
$$F_2 \leftrightarrow \exp\left(-\frac{D\Gamma^2/8}{(\omega-\omega_0-k\Omega)^2+\Gamma^2/4}\right), \qquad \phi_2-\phi_1 \leftrightarrow \frac{(\omega-\omega_0-k\Omega)D\Gamma/4}{(\omega-\omega_0-k\Omega)^2+\Gamma^2/4}. \tag{19}$$

It is significant that the quantities on the right side of (19) have an analogous sense, in the theory of resonant AM, to the corresponding numbers on the left side of (19), participating in the theory of non-resonant AM: τ/T is the effective relative contribution from the transmission through the medium 1, F_1^2 and F_2^2 are the attenuation coefficients for the two different media, and $\phi_2-\phi_1$ is the phase difference. The off-resonant states of the vibrating modulator act as a completely transparent medium in the resonant AM.

Relations (19) form a basis for a unified description of the two phenomena, which may be performed by a generalisation of the theory of non-resonant AM to include a frequency-dependent refraction index, simply substituting (19) in (7). Important evidence that the results derived formally are consistent may be deduced by the fact that, using (19), the response of the resonant modulator is

$$F_2\,\mathrm{e}^{\mathrm{i}\phi_2} \leftrightarrow \exp\left(\frac{\mathrm{i}D\Gamma/4}{(\omega-\omega_0-k\Omega)-\mathrm{i}\Gamma/2}\right), \tag{20}$$

which reveals the well known formula (Lynch *et al* 1960) for the change of a frequency component of the radiation, when it passes through an absorber possessing a set of resonant frequencies at $\omega=\omega_0+k\Omega$.

The residual difference between (4) and (14) is

$$\frac{\sin^2\{k\pi[1-\mathrm{J}_0^2(\varkappa A)]\}}{k^2\pi^2} \neq \mathrm{J}_0^2(\varkappa A)\mathrm{J}_k^2(\varkappa A). \tag{21}$$

However, if (21) is summed over k, one obtains, using (9),

$$\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1-\cos[2k\pi \mathrm{J}_0^2(\varkappa A)]}{2k^2} = \mathrm{J}_0^2(\varkappa A)[1-\mathrm{J}_0^2(\varkappa A)] = \mathrm{J}_0^2(\varkappa A)\sum_{k=-\infty}^{\infty}{}' \,\mathrm{J}_k^2(\varkappa A). \tag{22}$$

So, the non-equality (21) is explainable as a redistribution of the sideband intensities in the two approaches to the resonant AM (substituting (19) into (4), or using (14) directly). This redistribution is due to the different time-dependence of the modulation function, since a sinusoidally vibrated resonant modulator cannot produce a rectangle modulation on the radiation. At the same time, equation (22) confirms the condition that the sum of the sideband intensities must be the same, since it depends on the average characteristics (F_1, F_2, $\phi_2-\phi_1$, τ/T) of the modulation function only.

In the framework of the interpretation proposed here, an explanation of the amplitude modulation of the gamma radiation by a diffraction from a moving grating is not needed, since this conception could not be related to the case of the resonant AM. The argument of Cranshaw (1972) that the probability for detection of a gamma ray is independent of time, due to the large dimensions of the source and the detector compared to the grating constant, does not account for the fact that the experimentally observed spectrum is formed by multiplying the interferent picture of the partial waves, created by a sporadic photon.

References

Abragam A 1964 *L'Effet Mössbauer* (London: Gordon and Breach) p 22
Asher J, Cranshaw T E and O'Connor D A 1974 *J. Phys. A: Math., Nucl. Gen.* **7** 410
Cashion J D and Clark P E 1979 *J. Physique Coll.* **40** C2–44
Cranshaw T E 1972 *Phys. Lett.* **39A** 296
Hauser U, Neuwirth W and Thesen N 1974 *Phys. Lett.* **49A** 57
Isaak G R and Preikschat E 1972 *Phys. Lett.* **38A** 257
Kamenov P S 1970 *Rev. Roum. Phys.* **15** 1127
Lynch F J, Holland R E and Hamermesh M 1960 *Phys. Rev.* **120** 513
Mössbauer R L and Wiedeman W H 1960 *Z. Phys.* **159** 33
Ruby S L, Preston R S, Skov C E and Zabrausky B J 1973 *Phys. Rev.* **A8** 59
Tsankov L T 1980 *J. Phys. A: Math. Gen.* **13** 2959–67
Wojtowiecki W K and Sazonov S B 1975 *Proc. Int. Conf. Mössb. Spectr. Poland-Cracow, 25–30 August*, vol 2 p 399